Math 55 Quiz 2 DIS 105

Name: ______________________________ 7 Feb 2022

1. Let m and n be integers. Suppose that m is an odd number. Prove that mn is an even number if and only if n is an even number. [6 points]

   There exists an integer k such that $m = 2k + 1$.
   Suppose n is even. Then there exists an integer l such that $n = 2l$. Then $mn = (2k+1)(2l) = 2(2kl + l)$, where $2kl + l$ is an integer, hence mn is even.
   Suppose n is odd. Then there exists an integer l such that $n = 2l + 1$. Then $mn = (2k + 1)(2l + 1) = 2(2kl + k + l) + 1$, where $2kl + k + l$ is an integer, hence mn is odd. By contraposition, if mn is even then n is even.

2. Prove or disprove that for all sets A, B,

   (a) $A \cap (B - A) = \varnothing$ [2 points]

   (b) $\overline{A \cup B} = \overline{A} \cup \overline{B}$ [2 points]

   (a) This is true. Assume there exists an element $x \in A \cap (B - A)$. Then $x \in A$ and $x \in B - A$. But $x \in B - A$ means that $x \in B$ and $x \notin A$, the latter contradicting $x \in A$. Hence there cannot be any elements in $A \cap (B - A)$; in other words, $A \cap (B - A) = \varnothing$.

   (b) This is false. Suppose $U = \{1, 2, 3\}, A = \{1, 2\}, B = \{2, 3\}$. Then $\overline{A \cup B} = \varnothing \neq \{1, 3\} = \overline{A} \cup \overline{B}$.